Quiz 8

Thursday, June 9, 2016 4:55 PM

1) Is $\vec{F}(x,y,z) = \hat{i} + \sin z\,\hat{j} + y\cos z\,\hat{k}$ conservative ?

$$\text{curl } \vec{F} = \nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 1 & \sin z & y\cos z \end{vmatrix}$$

$$= \hat{i}\left[\frac{\partial}{\partial y}(y\cos z) - \frac{\partial}{\partial z}(\sin z)\right] - \hat{j}\left(\frac{\partial}{\partial z}(1) - \frac{\partial}{\partial x}(y\cos z)\right) + \hat{k}\left(\frac{\partial}{\partial x}(\sin z) - \frac{\partial}{\partial y}(1)\right)$$

$$= (\cos z - \cos z)\hat{i} - (0-0)\hat{j} + (0-0)\hat{k} = 0$$

- $\vec{F}$ is a vector field is defined on all of $\mathbb{R}^3$ whose components have continuous partial derivatives and $\nabla \times \vec{F} = 0$, and therefore $\vec{F}$ is conservative.

2) Show that the vector field $\vec{F}(x,y,z) = xz\,\hat{i} + xyz\,\hat{j} - y^2\hat{k}$ cannot be written as the curl of another vector.

- If $\vec{F} = \nabla \times \vec{G}$ for some vector field $\vec{G}$, then $\nabla \cdot (\nabla \times \vec{G}) = 0$.

However, $\nabla \cdot \vec{F} = \frac{\partial}{\partial x}(xz) + \frac{\partial}{\partial y}(xyz) + \frac{\partial}{\partial z}(-y^2)$

$$= z + xz - 0 = z + xz$$

Since $\nabla \cdot \vec{F} \neq 0$, we see that $\vec{F}$ cannot be the curl of another vector field.